Business 33911 - Economics of Information Final: Modeling Intertemporal Crime Reporting and Defense Spending Behavior Under Two ID Systems

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1 Introduction

Consider a market of townspeople and mafia. This is an intertemporal model with there are m townspeople and n mafia. I will use villager, townspeople, and civilian interchangably, as well as mafia, criminal, and robber. In every period, the townspeople receive W dollars. They are acting to minimize their losses in an intermporal choice model. These losses are incurred when individual mafia rob their house, stealing all W dollars. The mafia always succeed in stealing from the houses they target. However, the mafia has a chance of being caught, and conditional on being caught, thrown in jail for the duration of the game. Thus in the next period, if criminals are succesfully caught and jailed, there will be fewer criminals to steal from the townspeoeple. Actions that the townspeople can take involve purchasing defense d that increases the chances of the criminals being caught and reporting the crime, increasing both the chances of the criminal being caught and jailed. Under a real ID system, where each report IDs the reporter and calls them into court to testify, reporting is a costly act. However, compared to the anonymous system, reporting has much more impact on the criminal being jailed. Thus, jailing criminals is beneficial as it reduces the mafia in the next period and subsequently reduces the chances of being robbed. However, the cost paid is in defense choice and the reporting of behavior under the real ID system.

Using a simplified model, we have derived some important conclusions. With the assumptions of mafia and townspeople homogenity in actions, the assumption of constant mafia attacks, the assumptions on two period defense and reporting choices, and the assumptions regarding the shape of functions regarding probability of capture and probability of being jailed, we have grasped some conclusions. By comparing all permutations of ID system choice, reporting choice, and defense spending, through loss minimization in two time periods one can identify the dominant strategy the townspeople should collectively take to reduce their losses. We note that these relationships are based on the value of removing criminals and the cost of reporting. Notably, defense spending increases with expected variables such as discount rate, number of criminals, and initial wealth. More broadly, we have developed a system in which behavior can be revealed through a series of relationships between probabilities and expected values, allowing us to predict townspeople and mafia behavior. Though not empirically tested, these conclusions hit at the heart of these costly actions in free-riding situations such as this reporting and robbery system,

and a better understanding of the dynamics of these systems is reached.

2 The Town of Salem: A Model of Townspeople and Mafia

Suppose there is a town called Salem with m townspeople and n mafia members. Mafia memebers can steal money from the townspeople, and the townspeople can build defenses and report thefts to prevent future robbery. In each period, the actors conduct the following behavior in the following order. The townspeople are endowed with a certain amount of wealth. They then decide individually how much to spend on their own defense, which increases the chance that if they are robbed, the robber will be caught. Then, the mafia individually decide which houses to steal from, based on the no noise observable defense. If all defenses are the same, they randomly pick a house to rob. Each mafia member only robs one house. The townspeople witnesses these robberies taking place, but are powerless to stop them. They can, however, report the crime to the police. In the real ID system, they would incur a cost to report, but in the anonymous system, reporting is a costless action. The robbers then have a chance to get caught by the police, which is a function of the quality of defense and if they were reported. If they are caught, they are persecuted by the police. The probability of the robber subsequently going to jail depends on another function, which depends on whether or not they were reported and whether or not the real ID system is in place. If they are jailed, then that robber is removed from all subsequent periods. The game then repeats, with the jailed mafia leaving the game. During the game, all information is known to all parties, and all parties know that all parties know all the information.

Let us define the parameters of the game using mathematical language. The wealth endowment given to townspeople is W, their spending on defense is d, a continuous variable bounded by above by W and below by 0, and their discount rate is β. The decision to report is noted as $R \in \{0, 1\}$, with $R = 0$ indicating not reporting, and $R = 1$ indicates reporting. The reporting system is noted as $i \in \{0, 1\}$, where $i = 0$ is anonymous reporting, and $i = 1$ is using the real ID system. In the $i = 1$ state, the cost of reporting is the constant C. We assume that all townspeople are risk-neutral, have the same endowments and discount rates, and are otherwise all identical. This greatly simplifies the problem as it allows us to understand the group as an individual minimizing the same loss function, though as we shall see, the population size does

matter.

The mafia have the following associated actions and notations. The probability they are caught is noted as $p(d, R)$, a function of the defense spending of the house they attempted to rob and whether or not they were reported (a binary variable). If they are caught, the chance they go to jail is noted as $j(i, R)$, which is a function of whether or not they are in the real ID system and whether or not they were reported. One can combine these functions to find the probability of being jailed, which is noted as $g(d, R|i)$. The ID system is placed as a conditional, as the function is dependent on the ID system in place. All of these functions are bounded by $[0, 1]$ as is typical with probabilities.

3 Model Simplifications and Assumptions

While there are many other possible ways to model this situation and behavior, we have chosen this as the model that best represents reality while hittting at the heart of the tradeoffs and cost-benefit analysis that is associated with the choice of real ID reporting versus anonymous reporting. The following assumptions are key to the model:

Assumption 1: $p_d(d, R) > 0$

In other words, the more defense spending, the higher the chance of the mafia being caught and jailed. For example, defense spending could be loud alarms, which then allows the police to chase and confirm the criminals swiftly and make them more likely to caught the offender. This seems like a reasonable assumption, and thus gives incentive to purchase defense systems to deter criminals. Note that defense does nothing to prevent criminals from choosing to rob, but rather is beneficial for removing criminals from society.

Assumption 2:

$j(R = 1, i = x) > j(R = 0, i = x)$

and

$j(i = 1, R = 1) > j(i = 0, R = 1)$

and

$j(i = 1, R = 0) = j(i = 0, R = 0)$

These assumptions are regarding what affects the chances of being jailed. The first assump-

tion assures that being reported is a positive factor in the decision to jail a criminal. The power of reporting is not only in helping catch a criminal, but also can be used as evidence in the court to support imprisonment. This is true regardless of ID system. The second assumption says that if the criminal is reported, then under the real ID system, the chances of being jailed are much higher than under the anonymous system. This reflects the fact that appearing in court and formally accusing the criminal is much more effective in jailing the criminal than a statement of an anonymous report. This is necessary as otherwise, there is no benefit to the real ID system. The third assumption says that if there is no report, then the real ID system does not affect the chances of being jailed. This seems simply true.

More simplifications involve the way both the townspeople mafia are treated as individuals in one entity. Namely, that all decisions are identical for each member of each group. For the criminals this makes intuitive sense, as if all houses contain the same wealth and defense system, if one mafia decides to rob a house then all other mafia face the same problem and behave in the same way. Indeed, the villagers would make the same choices regarding defense and reporting, with the assumption that all villagers see all crimes occuring and have a choice to report. However, there are problems in making this assumption for the villagers.

In considering how much wealth to spend on defense, the villagers face a negative externality problem. If one villager spends more on defense, then not only are they reducing their own chance of being attacked (to 0), they are increasing the chance that the other villagers will be attacked. Thus, all other villagers would increase their spending in accordance. There is always a constant incentive to deviate and spend ϵ more to reduce attack chance to (0). Thus this is a form of prisoner's dilemma. The result is a Bertrand equilibrium, with all assumptions (equality of cost and product d) held. This means that the quantity of defense spending is the same for all townspeople, and is determined by equality of marginal product (in this case, value of defense) versus marginal cost (expected value of loss in a theft). This also means that the "buyers," the mafia, will split their discrete targets across with probability $\frac{n}{m}$ that each villager is targetted.

A more problematic assumption is that all the villagers will show the same reporting behavior. Though there is an incentive to report crimes, namely to jail criminals, reporting under the real ID system is costly to the individual. If all villagers are identical, if one villager would report, then all villagers would face the same problem and also report. However, a free rider problem develops as additional reports do not affect the chance of jail, so all townspeople would want to

deviate and not report as long as one report is submitted. This prevents a Nash equilibrium. The way we can get around this is to assume that if all villagers report, then one of them is randomly selected with a $\frac{1}{m}$ chance of bearing the full cost C of reporting. Thus, the expected cost of reporting is $\frac{C}{m}$ and all townsfolk report.

Another major simplification is that we ignore any maximization of profits of the mafia. Ideally, the mafia would make a decision each period of whether or not to attempt robbery, given the stealable wealth and the probability of them being caught given defenses. However, this would greatly complicate the model, as it would become a game theoretical leader-follower problem. It is far simplier to assume that robbers will always attempt to rob the villagers and reduce the problem to villagers making decisions.

The final simplification involves using two time periods instead of an infinite horizon. Ideally, each period would involve decisions about defense spending, using the ID system, and other parameters, given that n changes as each time period passes. In addition, wealth would carry over instead of being endowed at each period. We have chosen instead to only focus on spending on defense in the first period, which influences criminals jailed and thus, the likelihood of being robbed in the second period. Thus, the villager does not spend any money on defense in the final (second) period. This simplification still captures most of the interesting elements of the situation. It is still intertemporal and involves the cost-benefit analysis between using the ID system and reporting to remove criminals.

With the knowledge of the game and its various variables, we can set up the villager two-period maximization problem.

4 Analysis and Results

The villagers face the following loss minimization problem:

$$\begin{aligned} \underset{d,R,i}{\text{minimize}} \quad & L(d,R,i) = \frac{n}{m} * W + d + \frac{C}{m} * (R|i) + \beta * (\frac{n-a}{m} * W) \\ \text{where} \quad & a = p(d,R) * j(R,i) * n = g(d,R|i) * n \end{aligned} \tag{1}$$

The first term $\frac{n}{m}$ is the probability of being robbed when mafiosos choose at random which house to rob, given that all defenses are the same. Thus, $\frac{n}{m} * W$ is the expected loss. $\frac{C}{m}$ is

the cost of reporting, where $R = 0$ is no reporting and thus no cost, $R = 1$ is reporting, and $R = 1|i = 0$ is also no cost. The expression after β is the next period's expected loss. The probability of being robbed in this period is smaller, as a number of criminals will have been caught and jailed, which depends on the d, r, i parameters and the total population of criminals, n. Technically there are no incentive or other constraints as each villager will always play the "townspeople and mafia" game. They are simply minimizing losses they will face from defense spending, being robbed, and potentially reporting.

What we are interested is in whether or not the townspeople will support the real ID system that places a cost on reporting. Thus, we must compare the following expressions: $L(d_1^*, R = 1, i = 1)$, $L(d_2^*, R = 0, i = 1)$, $L(d_3^*, R = 1, i = 0)$

or more simply:

$$\min_{d^*,R,i} \arg \quad L(d^*, R, i), \qquad R, i \in \{0, 1\} \tag{2}$$

where $d*$ solves the previous minimization problem. Note that the fourth condition, $R = 0, i = 0$ is omitted. This is derived from the three assumptions about jail probability, as reporting in the anonymous system is free and only increases the chance of criminals being caught and jailed, which then decreases the number of criminals in the next period.

So in order to solve these equations, we can use the following approach. First, find

$$\min_{d^*,R|i=1} \arg \quad L(d^*, R, i = 1), \qquad R \in \{0, 1\} \tag{3}$$

to find the behavior if the real ID system is pursued. This is done by solving the loss minimization problem outlined in (1), and then plugging the values of d^* into the loss function and obtaining an expression. The expressions obtained from $R = 1$ and $R = 0$ are likely different, with the smaller value being the dominant behavior for the townspeople. Then, to solve (2), one must compare the previous loss function in state $i = 1$ with $L(d^*, R = 1, i = 0)$. The lowest function out of those two then is the chosen behavior of the villagers, as it minimizes the two period losses the best.

To solve for d^* in these cases, we will take the first order condition with respect to d. For case 1, where $R = 0, i = 1$, the first order LaGrangian is as follows:

$$1 - \frac{n}{m} * \beta * W * p_d(R = 0, d^*) * j(R = 0, i = 1) = 0$$

Solving for expressions including d^* yields:

$$p_d(R = 0, d^*) = \frac{m}{n} * \frac{1}{W * \beta} * \frac{1}{j(R = 0, i = 1)}$$

This is rather hard to read. Let's place some functions in for $p(.)$ and $j(.)$. Suppose that $p(R, d) = \frac{d}{d+1} * |R - \frac{1}{3}|$. Note that the previously mentioned assumptions about boundedness of this function and it increasing in d is held. Thus, the previous expression is

$$(d + 1)^2 = \frac{n}{m} * W * \beta * j(R = 0, i = 1) * \frac{1}{|R - \frac{1}{3}|}$$

$$d^* = \sqrt{\frac{n}{m} * W * \beta * j(R = 0, i = 1) * \frac{1}{|R - \frac{1}{3}|}} - 1$$

Note that this expression is very similar to the two other cases, as the other cases simply replace the $j(.)$ function. Without plugging in an expression for $j(.)$, we can draw some implications. We see that as wealth, proportion of mafia compared to villagers, discount rate, and chance of being jailed go up, then spending on defense also goes up. These are expected results, as all those increase the value that the villager places on removing criminals. Proportion as more criminals means higher marginal chance of being robbed, more wealth means more to lose if robbed, higher discount rate means higher value of not being robbed in the future, and higher jail probability meaning being caught is more likely to lead to mafia imprisonment and removal from the system.

The next step is to plug in this value of d^* into equation (1) which yields the following expression:

$$L(d^*, R = 0, i = 1) = \frac{n}{m} * W + \sqrt{\frac{n}{m} * W * \beta * j(R = 0, i = 1) * \frac{1}{|R - \frac{1}{3}|}} - 1 + \beta * (\frac{n - a}{m} * W)$$

We do the same process to find the other two cases, with the expressions being as follows:

$$L(d^*, R=1, i=1) = \frac{n}{m} * W + \sqrt{\frac{n}{m} * W * \beta * j(R=1, i=1) * \frac{1}{|R - \frac{1}{3}|}} - 1 + \frac{C}{m} + \beta * (\frac{n-a}{m} * W)$$

and

$$L(d^*, R=1, i=0) = \frac{n}{m} * W + \sqrt{\frac{n}{m} * W * \beta * j(R=1, i=0) * \frac{1}{|R - \frac{1}{3}|}} - 1 + \beta * (\frac{n-a}{m} * W)$$

In practice, we would compare these three expressions using (2) to determine the correct bundle of behavior, (d^*, R, i). This is rather difficult to do within the assigned space without inputting various values, but we will discuss the relationships between variables here theoretically.

The second expression $(R = 1, i = 1)$ includes the $\frac{C}{m}$ expression not found in the other choices. If this option is best, the chance of being jailed must be much higher when reporting versus not reporting in order in order to make up for this extra cost. The benefit of reporting would be to drastically reduce the number of criminals in the next period, and thus reduce the probability of being robbed. In a much more simplified example, if there were one criminal, and reporting him would ensure he was jailed, then the probability of being robbed would decrease by $\frac{1}{7}m$, and the expected loss would decrease by $\frac{1}{7}m * W$. Thus, if the cost of reporting is less than $\frac{1}{7}m * W$, all individuals would report the crime, with one unlucky individual bearing all the cost. Of course, d^* and other variables move outside of this relationship, but this simplified example highlights the trade-off between reporting cost and more jailed criminals.

5 Discussion

We have noted that defense spending goes up in relation to the variables of interest that increase the value of jailing criminals, such as discount rate and probability of being jailed if caught. Thus, we see the villagers correctly reacting to incentives regarding future wealth, considering that the only real reason to report criminals is to reduce the chance of being robbed in the future. While we were unable to fully develop a powerful expression that determines the choice of ID system, and reporting choices within that ID system, the framework and empirical steps to do so have

been thoroughly outlined in the paper. The key finding is that the benefit of reducing criminals in the second period must outweigh the cost of reporting. It is in this author's opinion that it is unlikely that there will be no reporting while choosing to use the real ID system, as the town might as well report anonymously, which increases the chances of criminals being removed. The choice between reporting within the real ID system and the choice of reporting without the ID systems lies on the cost of reporting, as well as the probabilities that reporting under the real ID system will provide a large enough benefit, keeping in mind the shifting value of d^*. Also, there appears to be an "optimal" amount of crime, in which the cost of reporting may be too great to want to jail criminals, at least under the costly real ID system. In the anonymous system, as well as the real ID albeit slower, the number of criminals will eventually dwindle down to 0.

The next step would involve empirically testing the relationship between these tradeoffs and specific variables, as well as producing an number of graphs and other charts highlighting these movements. Due to time and space consideration, these will be left to future authors. Indeed, while many other specifications can be made to this model, most forwardly invovling multiple time periods, wealth or risk differentiation amongst villagers, and looks into criminal profit maximization behavior, this simplified model has uncovered many of the problems inherent to costly behavior in free-riding situations, such as the reporting of criminals. Oftentimes the free cost option, though perhaps less effective, is less costly than a more effective paid solution, and vice versa.